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ABSTRACT

The Minnesota School Mathematics and Science Teaching (MINNEMAST) Project is characterized by its emphasis on the coordination of mathematics and science in the elementary school curriculum. Units are planned to provide children with activities in which they learn various concepts from both subject areas. Each subject is used to support and reinforce the other where appropriate, with common techniques and concepts being sought and exploited. Content is presented in story fashion. The stories serve to introduce concepts and lead to activities. Imbedded in the pictures that accompany the stories are examples of the concepts presented. This unit presents a fundamental geometric concept of rigid motion. Two types of simple motion are presented to help children to find what patterns are unchanged by these motions; the two forms are simple translatory symmetry or repeating patterns, and bilateral symmetry or mirror reflection. In the process, children discover a number of geometrical relationships. This unit will provide the initial formative concepts necessary for various operations with symmetry. Worksheets and commentaries to the teacher are provided and additional activities are suggested. (JP)

MINNE

Math And Science Teaching project

UNIT VI
SYMMETRY

MINNESOTA SCHOOL MATHEMATICS AND SCIENCE CENTER 1965

MATHEMATICS

FOR THE

ELEMENTARY SCHOOL

Unit VI

Symmetry

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MINNESOTA MATHEMATICS AND SCIENCE TEACHING PROJECT

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We are deeply indebted to the many teachers who
used earlier versions of this material
and provided suggestions for
this revision

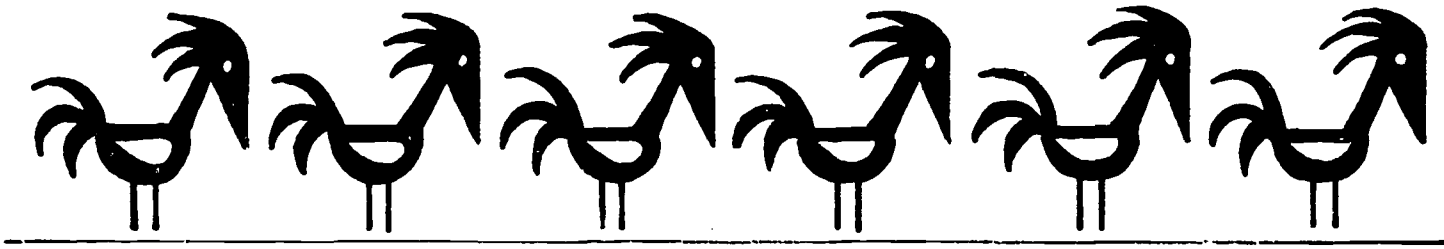
CONTENTS

Teacher Background on Symmetry

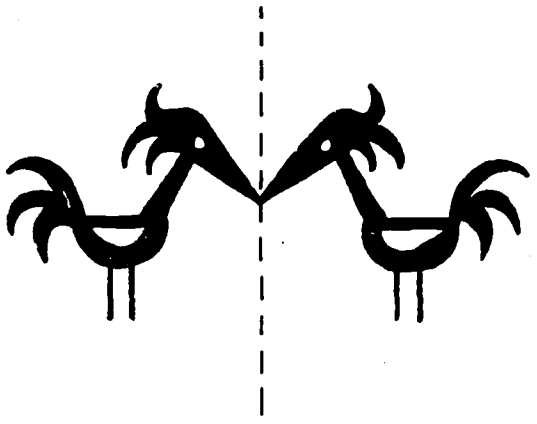
Symmetry is one of man's ways of attempting to comprehend and create disciplines, beauty and perfection.

The principle of symmetry may be applied in the arts and in inorganic and organic nature. Symmetry has philosophical and mathematical significance. The essential concepts of symmetry were first developed from their applications in mathematics.

In this unit the children will study a fundamental geometric concept of rigid motion, that is, a motion with distances unchanged. Two types of simple motion are presented to help the children to find what patterns are unchanged by these motions. In the process children discover a number of geometric relationships. The two forms are: simple translatory symmetry or repeating patterns, and bilateral symmetry or mirror reflection. This unit will provide the initial formative conceptions necessary for various operations with symmetry.



TRANSLATORY SYMMETRY (Repeating Patterns)



BILATERAL SYMMETRY (Mirror Reflection)

Reference is given to the following books for a more extensive definition and appreciation of symmetry:

Celebonovic, Stevan, The Living Rocks, Philosophical Library, 1957.

Feininger, Andreas, The Anatomy of Nature, Crown Publishers, 1956.

Jirovec, O., Life Under the Microscope, London: Spring Books, 1959. (Out of print.)

Strache, Wolf, Forms and Patterns in Nature, Pantheon, 1956.

Thompson, D'Arcy, Growth and Form, Cambridge University Press, 1952.

Weyl, H., Symmetry, Princeton University Press, 1952.

Pass out "Happy Helpers" to the class.

Read "Happy Helpers" to introduce the concept of repeating patterns.

The vocabulary in the story is somewhat controlled so that the children will be able to read the story to themselves later in the year. Encourage the children to add the story to their mathematics library.

HAPPY HELPERS

"Billy," said Mother, "I have so many things to do today. Will you help me with something?"

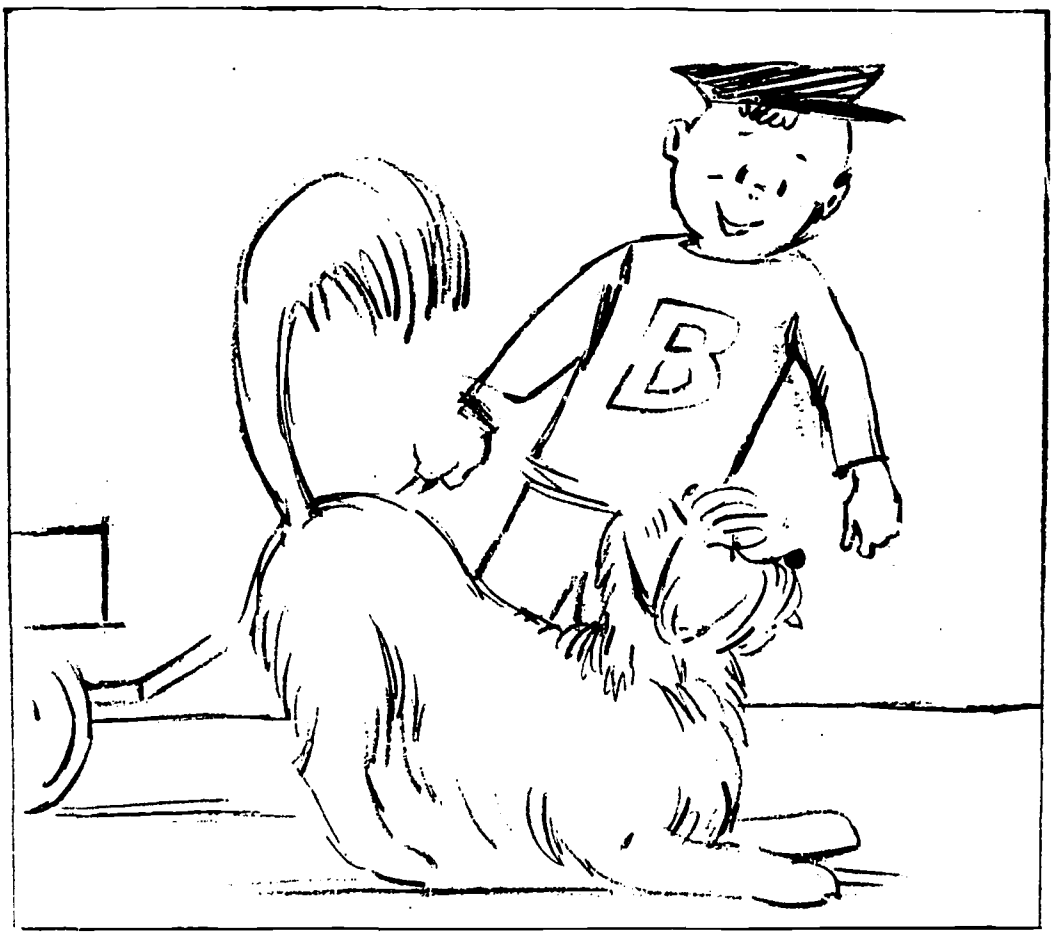
"What would you like me to do, Mother?" asked Billy.

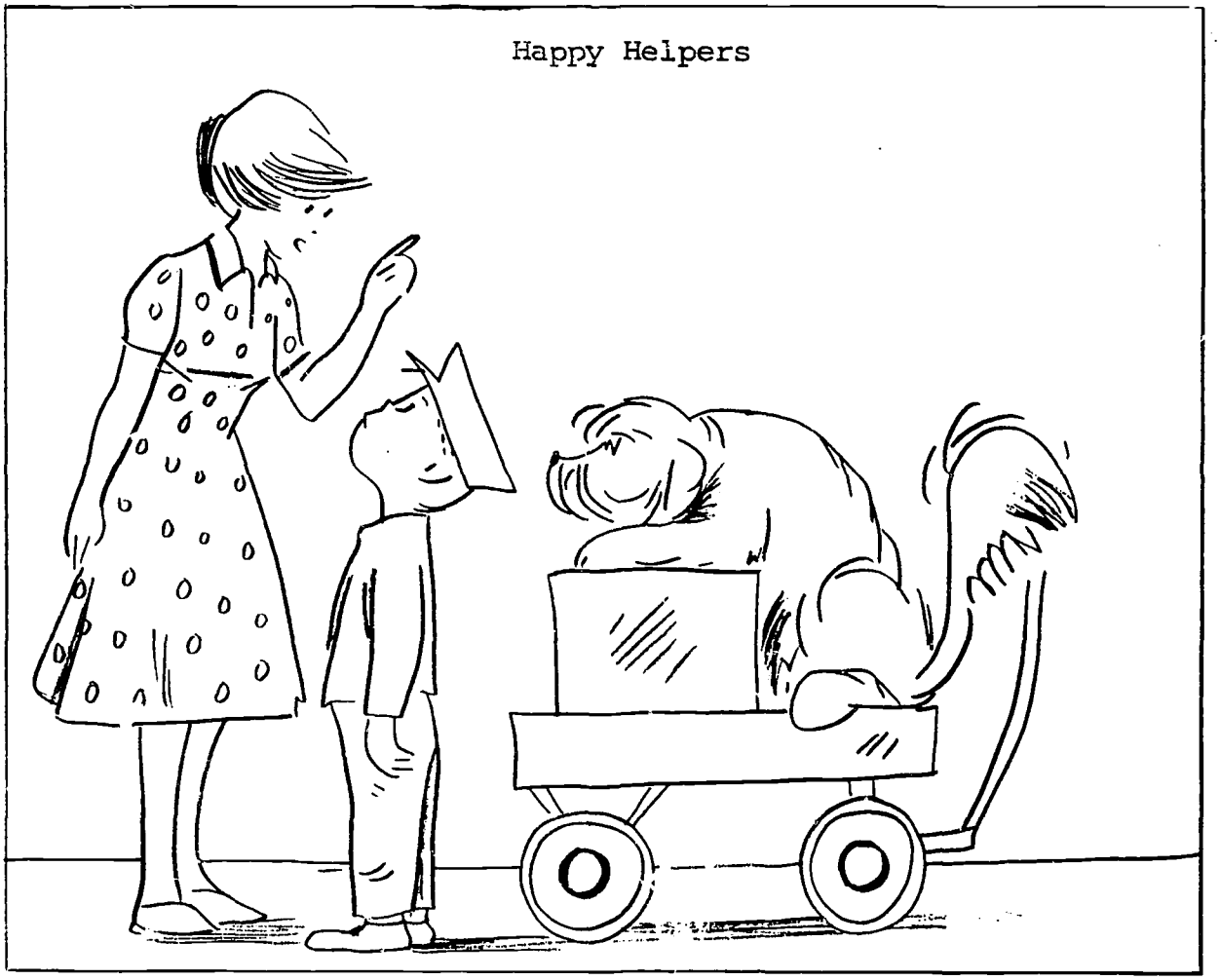
"Uncle John and Aunt Sally need this box of old rags. They will use the rags when they paint their new house. They just moved into a house on our street. It isn't far from here," said Mother.

"I know I can find their house," said Billy. "This is really going to be fun. I will put the box in my wagon. Today I will be a big delivery boy. May Skippy go, too?"

"Yes . . . I think you are both BIG BOYS now. Would you like to go with Billy, Skippy?"

"Bow wow! Bow wow!" said Skippy.





Mother laughed. "You have to know where you are going. I don't think that just your noses will get you there. You will have to use your eyes, too. This is what you will see in Uncle John's block.

First, you will see a red house.

Next, you will see a yellow house.

Then, you will see a big green tree.

Last, you will see Uncle John's blue house.

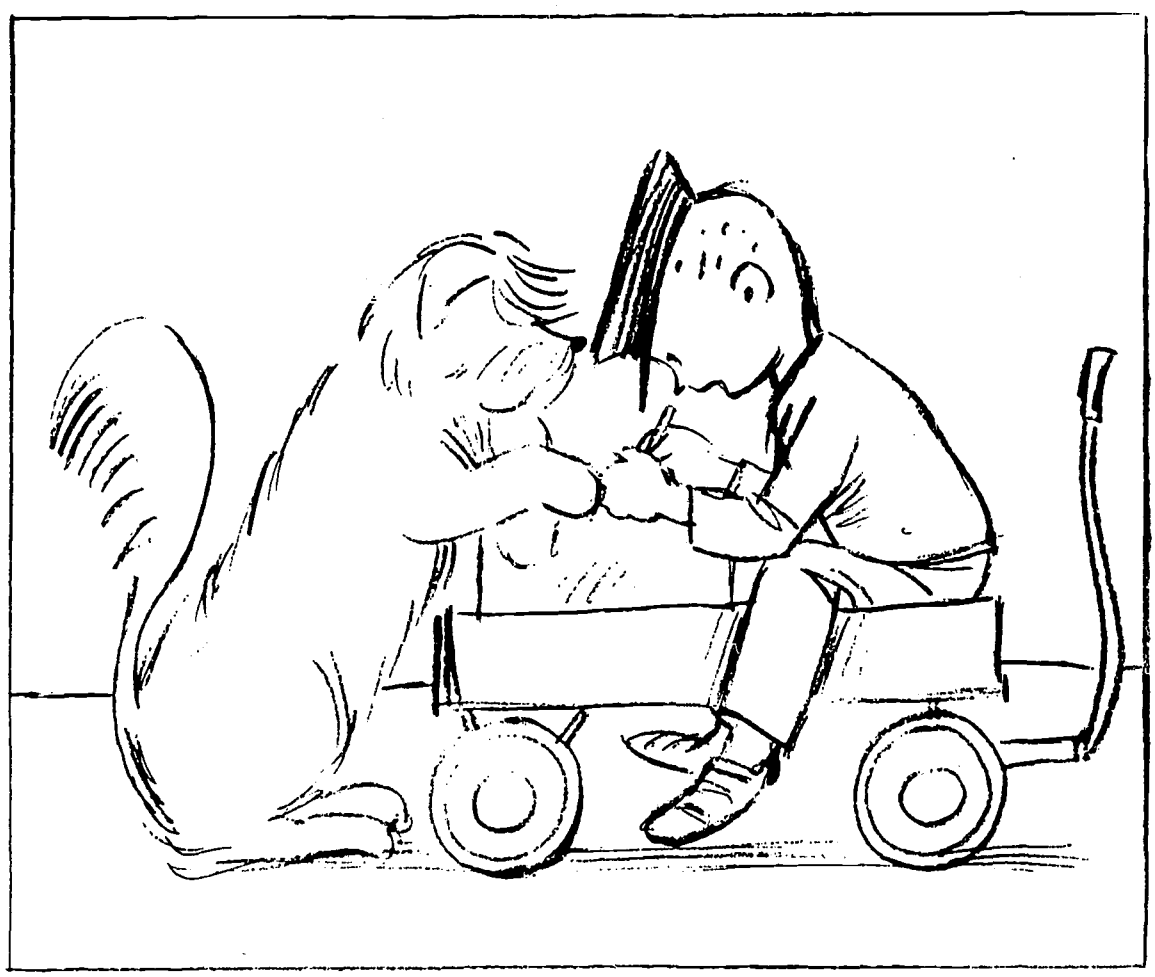
REMEMBER.... You have to go east on Happy Avenue."

Billy nodded, "Oh, that will be easy to find. I think I know where it is. Uncle John's house is the blue one on the corner."

"Yes," said Mother. "The address is....."

"I know. Good-by, Mother," shouted Billy. "Here we go, Skippy."

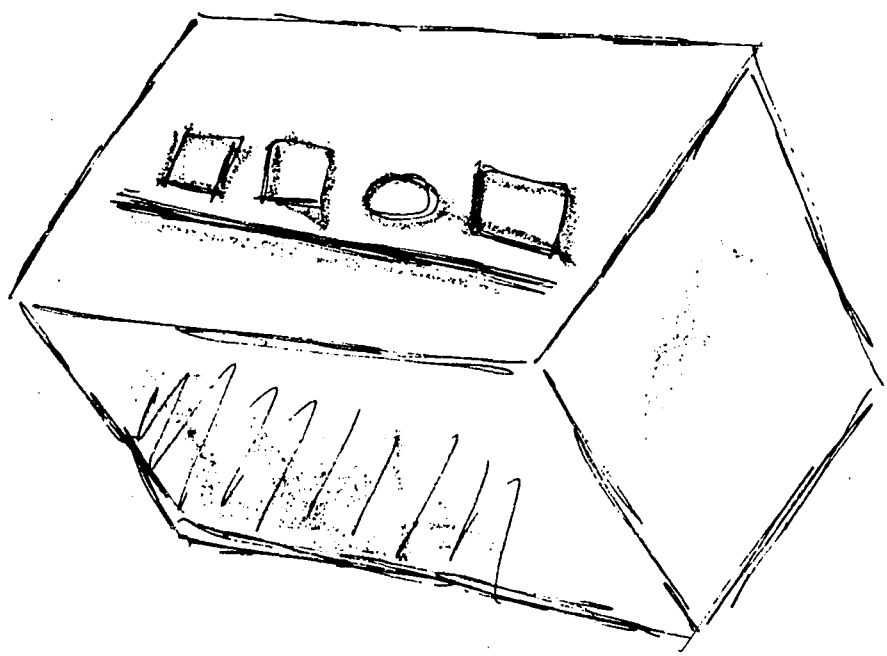
The big delivery boy and his helper, Skippy, didn't go far before Billy got an idea.



"Do you know what, Skippy? I think I should make a map . Then I will be sure to make the delivery to Uncle John. I will draw three houses. I will write R on the house that is red. I will write Y on the house that is yellow. What do you think I should write on the blue house?"

"Bow wow?" said Skippy.

This is part of the map Billy made on the top of the map box.

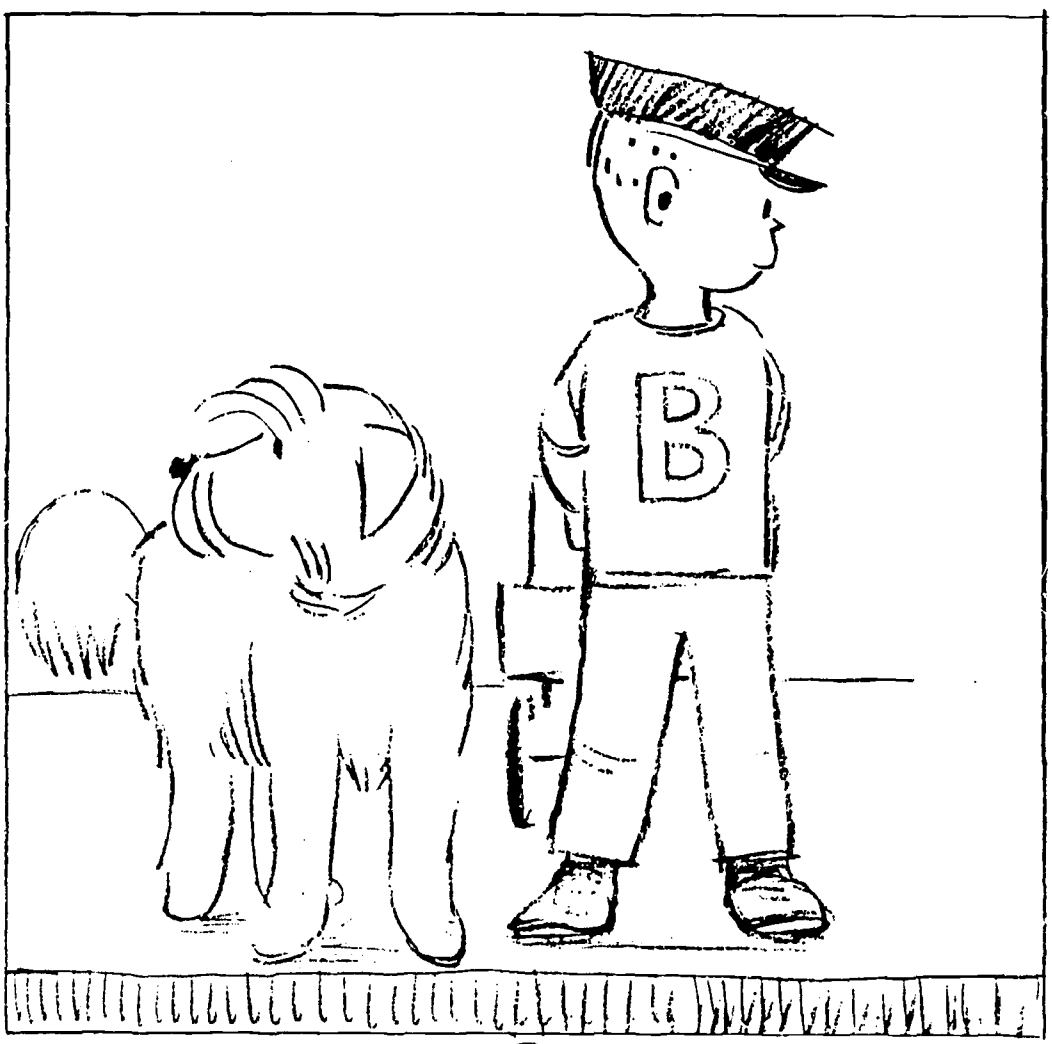


He made squares to represent the houses and a circle to represent the tree.

(Note to teacher: Write the letters in the squares to demonstrate how Billy did it. Begin at the left with the letter for the red house.)

Billy and Skippy began to walk east on Happy Avenue. Would they have to go far? Would they find Uncle John and Aunt Sally? They looked both ways and crossed the first street.

"Do you see what I see?" said Billy. "There is the red house. There is the yellow house. There is the big green tree. This must be Uncle John's house! It is blue."



Billy ran up to the house. He rang the doorbell. A little old lady came to the door. She had a pretty black and white cat with her.

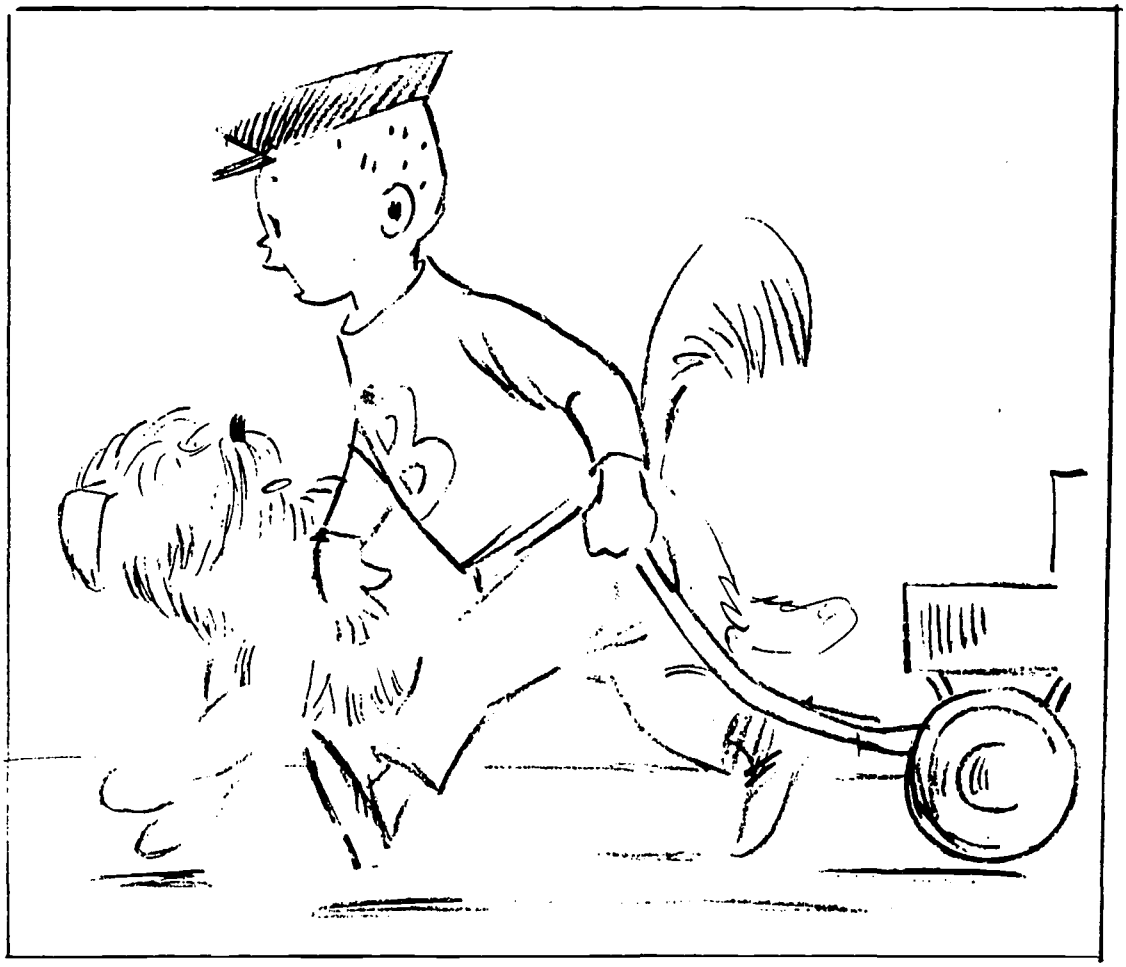
"Hello," she said.

"Hello, is my Uncle John here?" Billy asked.

The little old lady said, "No, he isn't here. You have the wrong house. I'm sorry."

"I'll have to keep looking, I guess. Thank you."

He and Skippy began to walk again. "I guess Uncle John lives in the next block. There is the first house. It is red. Good! The next one is yellow. There is the big green tree. Now we have found Uncle John's house!"

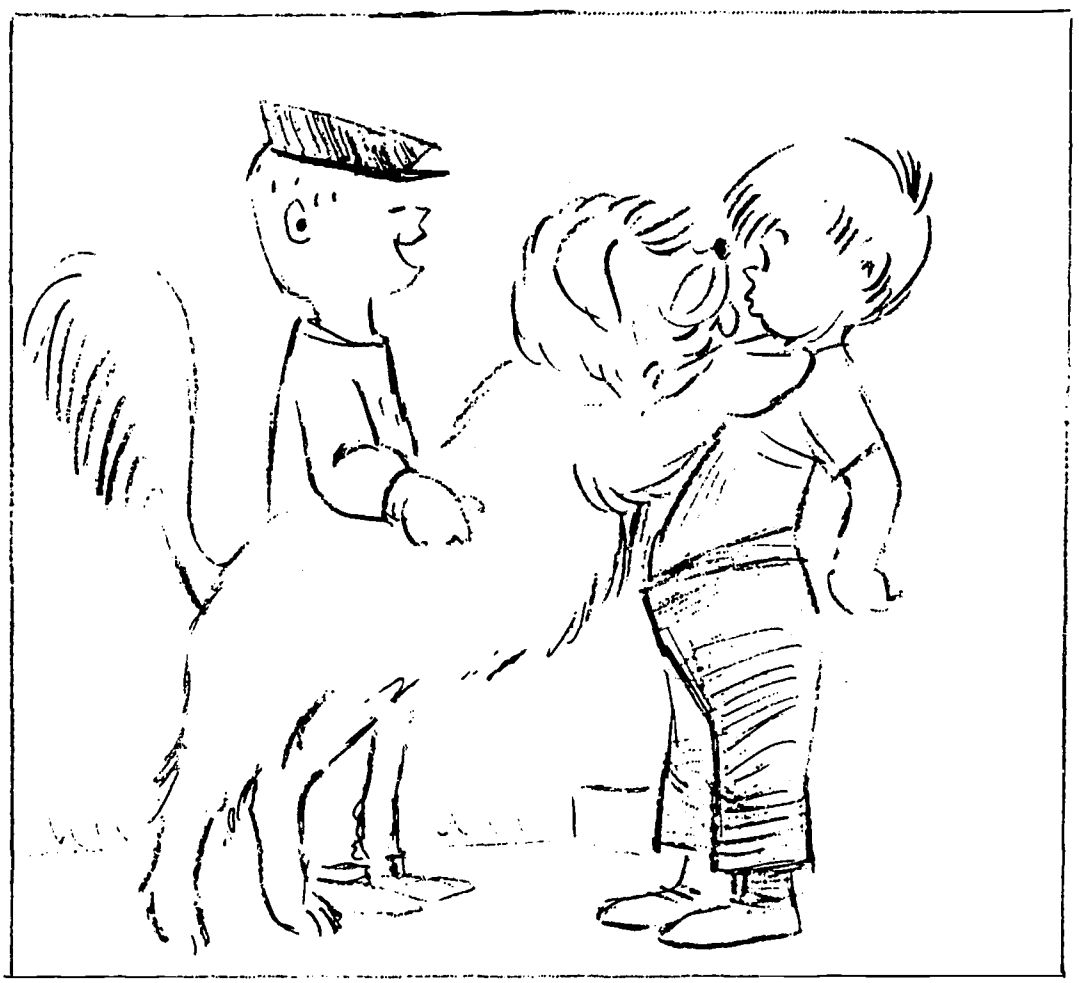


Billy ran up to the house and rang the doorbell. This time a little boy came to the door. The little boy said, "Hello, my name is Tommy. What is your name?"

"My name is Billy and this is my dog Skippy. Do you know where my Uncle John is?"

"Who is your Uncle John?" asked the little boy.

Billy laughed, "I guess he isn't here. Good-by, Tommy."

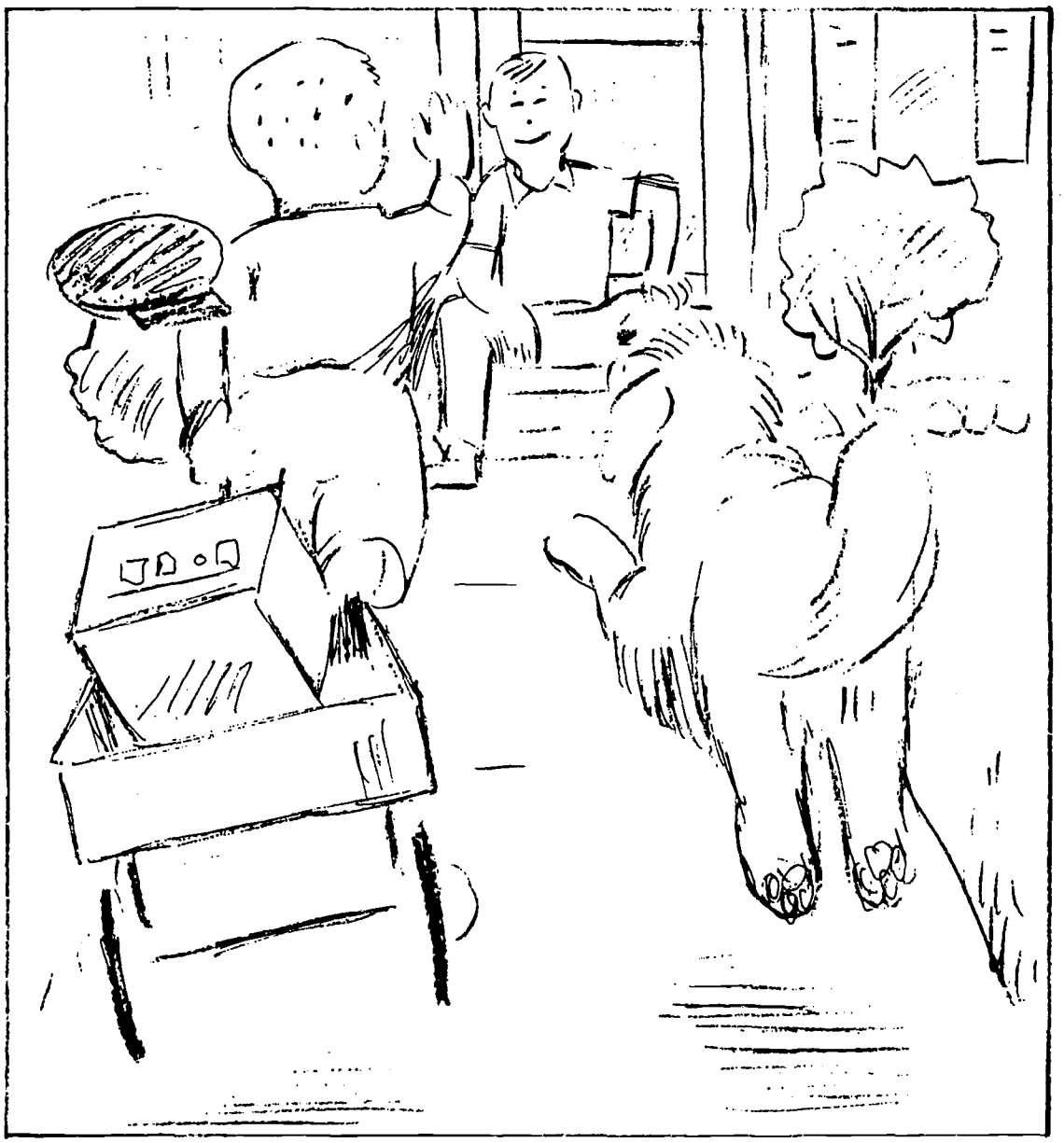


Billy looked at Skippy. He said, "This is funny. My map is right. Why is the blue house always wrong? I will keep looking. I know his house is on Happy Avenue."

"Look! Here are a red house, a yellow house, a tree, and a blue house. This may be it!"

Billy ran fast with Skippy right after him.

"Uncle John! Am I happy to see you! I made a map to help me find you. It didn't work too well. All the blocks on Happy Avenue look the same!"

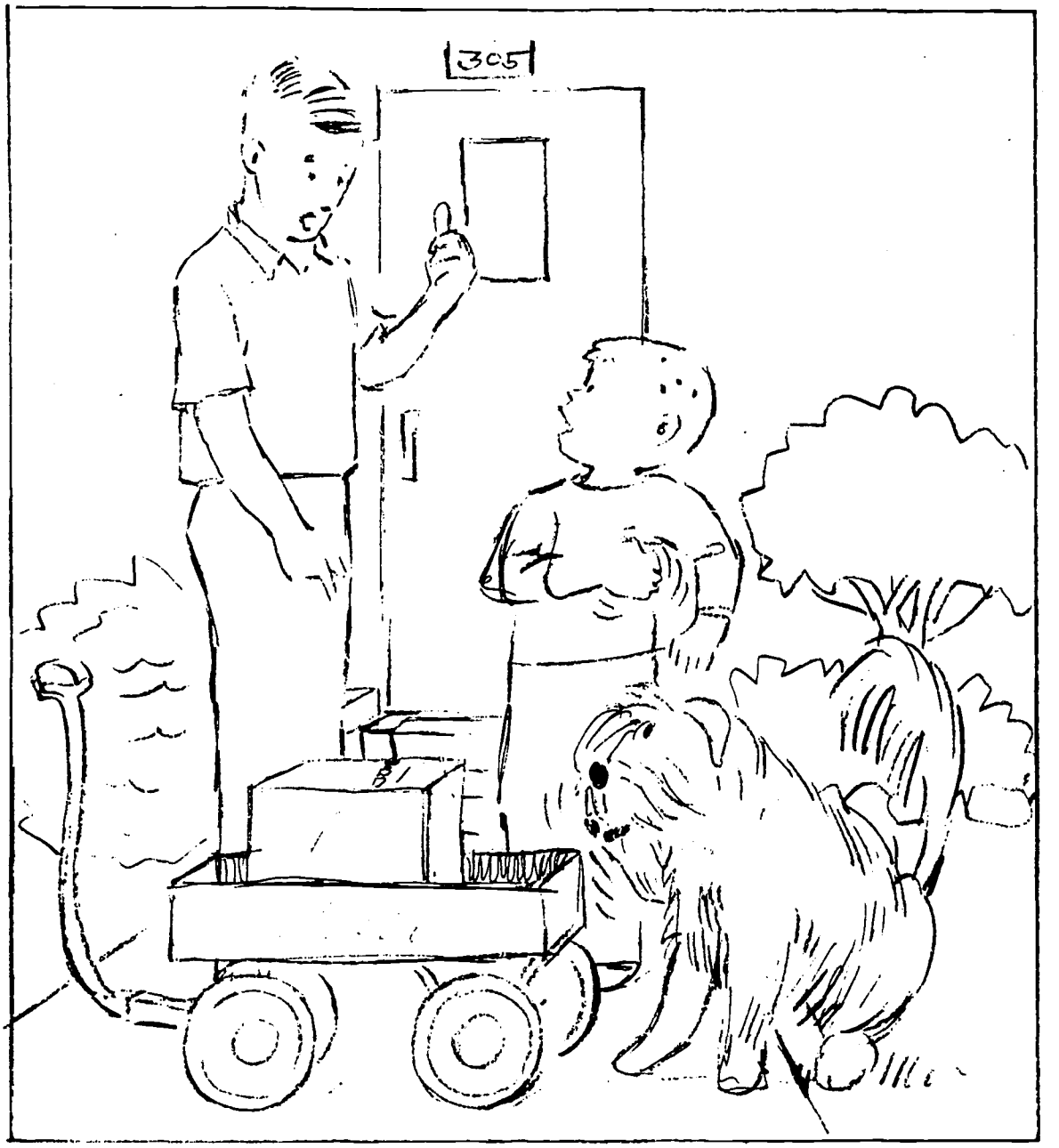


"Am I happy to see you!"

Uncle John looked at it and laughed.

"This map would be fine. It needs just one more thing on it." He pointed to the numbers above the door.

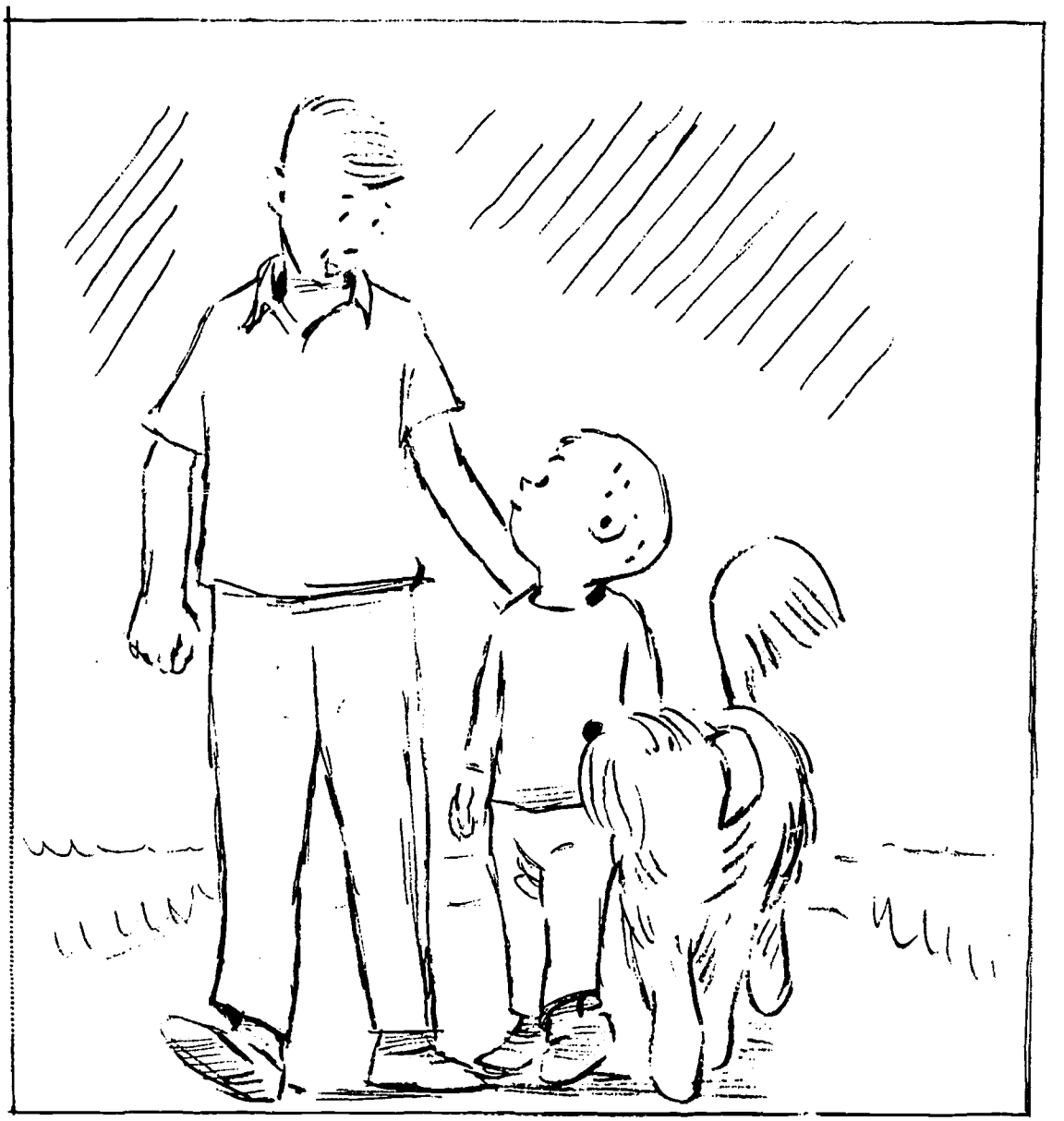
Billy smiled. "Mother wanted to tell me the address. Now I know why I needed the numbers."



"Come on in, Billy. Aunt Sally has something for you. I have something for Skippy. Besides I'd like to show you something."

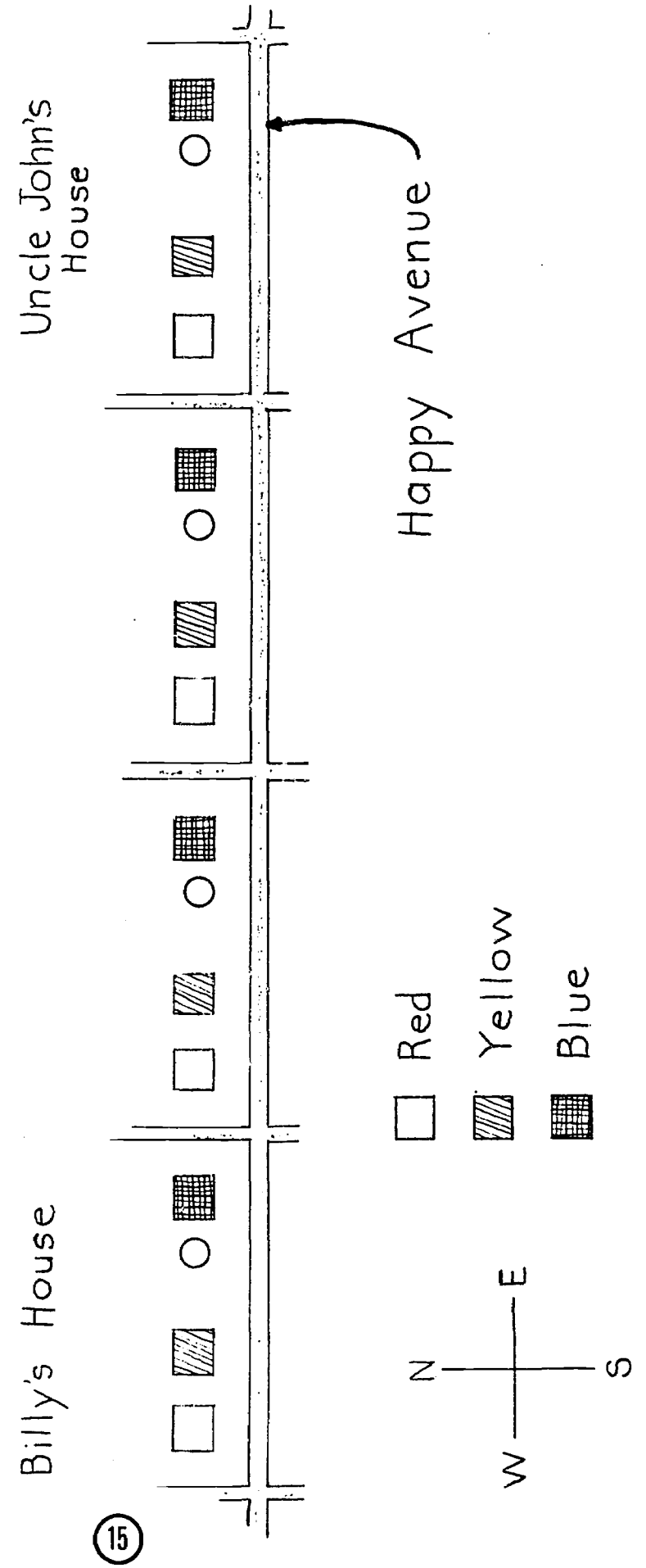
Uncle John took some paper and made a map.

He said, "Look at the map from your house to this house. The houses and tree make a pretty pattern."



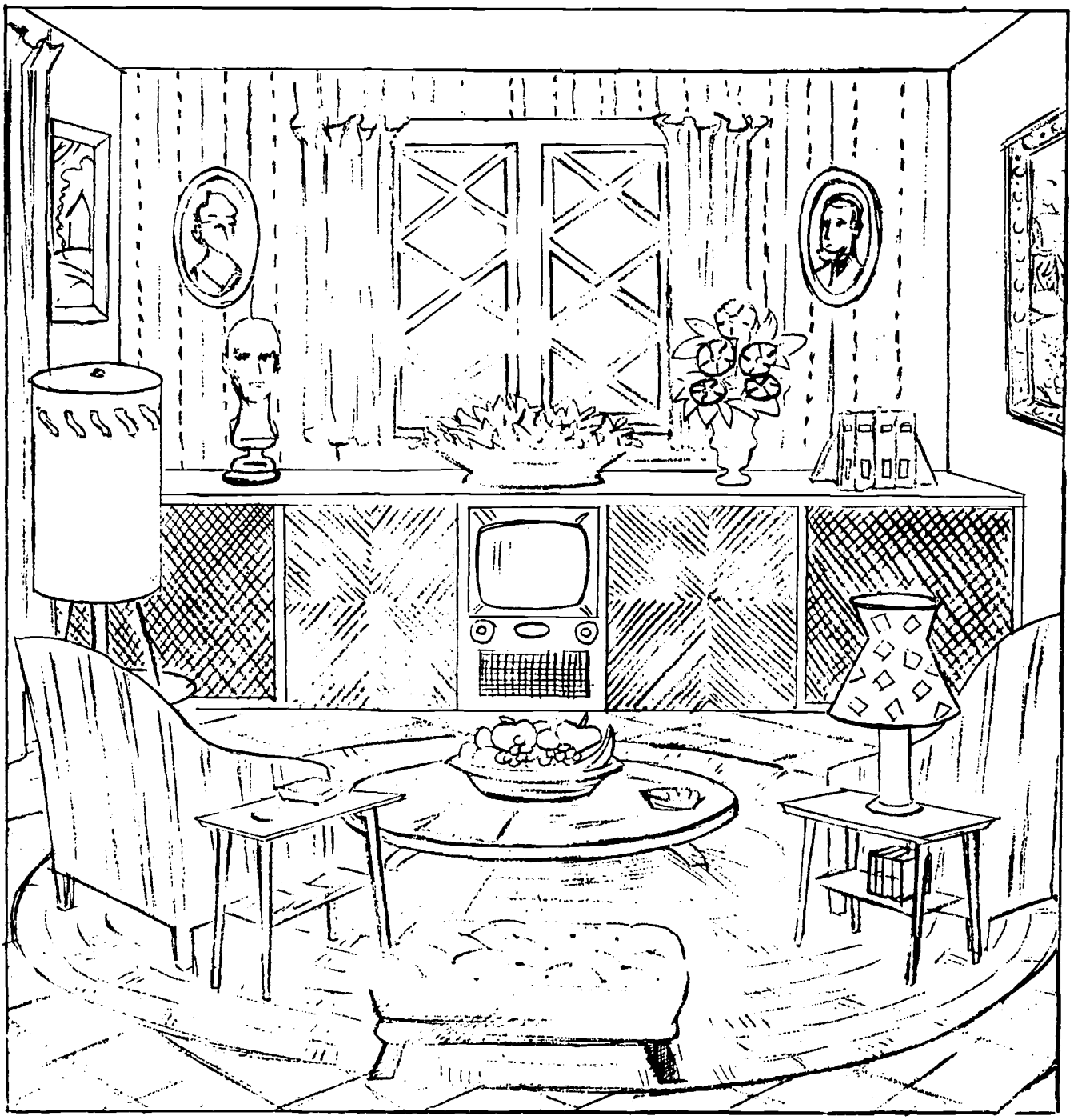
Here is the map Uncle John made. What did Uncle John do to show the patterns of the houses in each block?

Why did he do this?



Billy looked at Uncle John's map.

Then Uncle John said, "Look around the room. You can see many repeating patterns."



Billy saw one right away on the rug.

Billy saw many repeating patterns. He saw them on the cookies Aunt Sally gave! He even saw a pattern Skippy made with his paws.

"There are many repeating patterns on Happy Avenue!" he said.

Let's go back one page and look at the room again. How many repeating patterns can you find?

Do you see any repeating patterns in your classroom?

SIMPLE TRANSLATORY SYMMETRY

Teacher Background

Operational Definitions (to be used in classroom discussion):

Pattern: An example to be copied

Design: The arrangement of elements that make up a work of art

Repeating pattern: Translatory symmetry

Row: A number of objects in an orderly series

Trace: To copy by following lines as seen through a transparent sheet superimposed on the original

Design paper: The original design which is to be traced

Tracing paper: A transparent sheet of paper

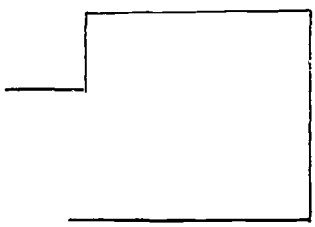
Suggested Activities

1. The class sings through "Looby Loo" with the corresponding motions as a review of left and right conception.
   "Looby Loo", Music For Young Americans, A.B.C. Music Series, Kindergarten, American Book Company, New York: 1959, p.83.

Commentary on Worksheet 1 - Design A - Repeating Patterns

Let's use our pointing finger as a pencil and trace the first pattern at the left in Design A. We'll start at the left and follow through: one across, one up, three across, three down, and three back.

Design A Pattern



Let's do it again with the next repeat of the pattern: one across, one up, three across, three down, and three back. (Repeat this process until the entire design has been traced.)

Now we're ready to use our pencils and tracing paper.

FIRST, place the tracing paper on the design paper so that all the edges are even.

NEXT, let's trace all of the patterns the way we did with our fingers so that the tracing paper patterns fit exactly on top of the design paper patterns.

QUESTIONS:

What directions can you move your tracing paper so that at least one pattern on the tracing paper fits on top of one pattern on the design paper? (Ans.: to the left and right.)

How far to the right can you move the tracing paper and still get a tracing paper pattern on top of the design paper patterns? (Ans.: To the last pattern on the right on the design paper.)

How far to the left can you move the tracing paper and still get a tracing paper pattern on top of the design paper patterns? (Ans.: To the last pattern on the left on the design paper.)

Note: A second experiment with Design A may be done if the need arises for further clarification of translation of sets of points. The same tracing paper may be used for this experiment by employing the guideline on the design paper.

Commentary on Worksheet 2 - Design B - Repeating Patterns

The same procedure is to be followed in utilizing Design B as was used in Design A.

Design B provides a translation of a simple open and closed curved line pattern to illustrate the fact that designs composed of different types of line forms may be translated.

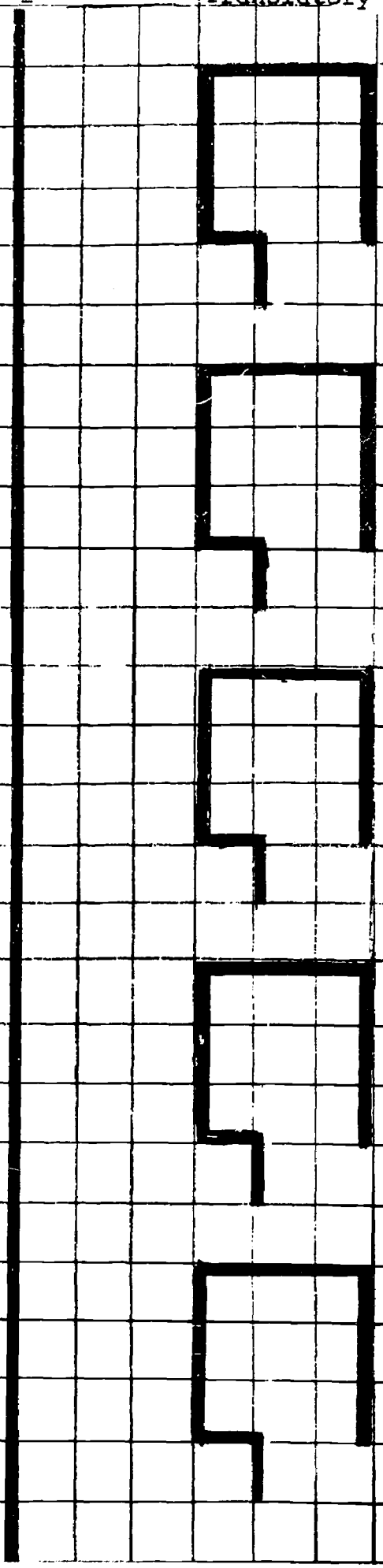
Commentary on Worksheet 3 - Design C - Repeating Patterns

Design C utilizes a combination of straight and curved lines visually organized to represent one of the favorite characters of young children, the happy clown.

It should be stressed that the first four design activities in this unit are to be considered experiences in translation of sets of points on graph paper rather than creative art experiences.

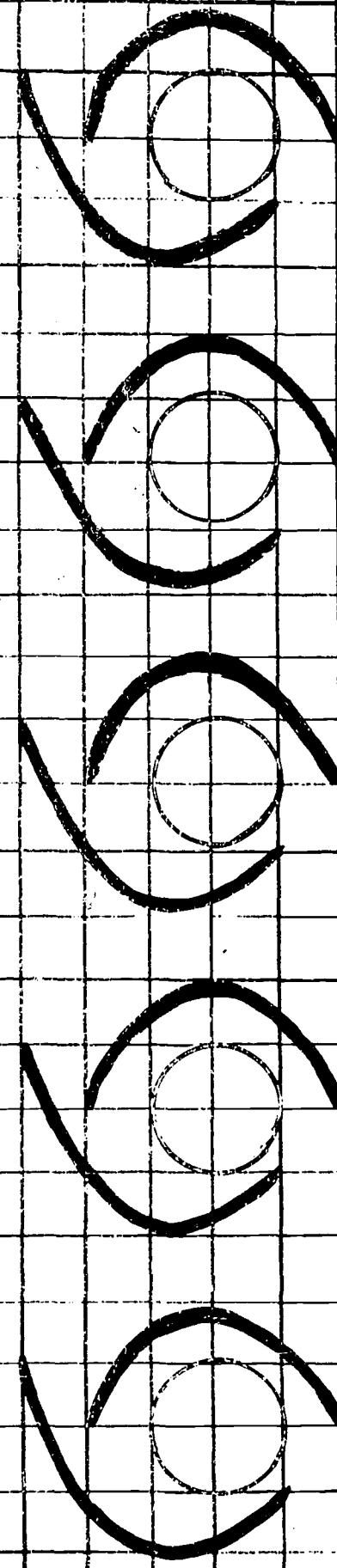
The children are to experiment with Design C on tracing paper.

Design A

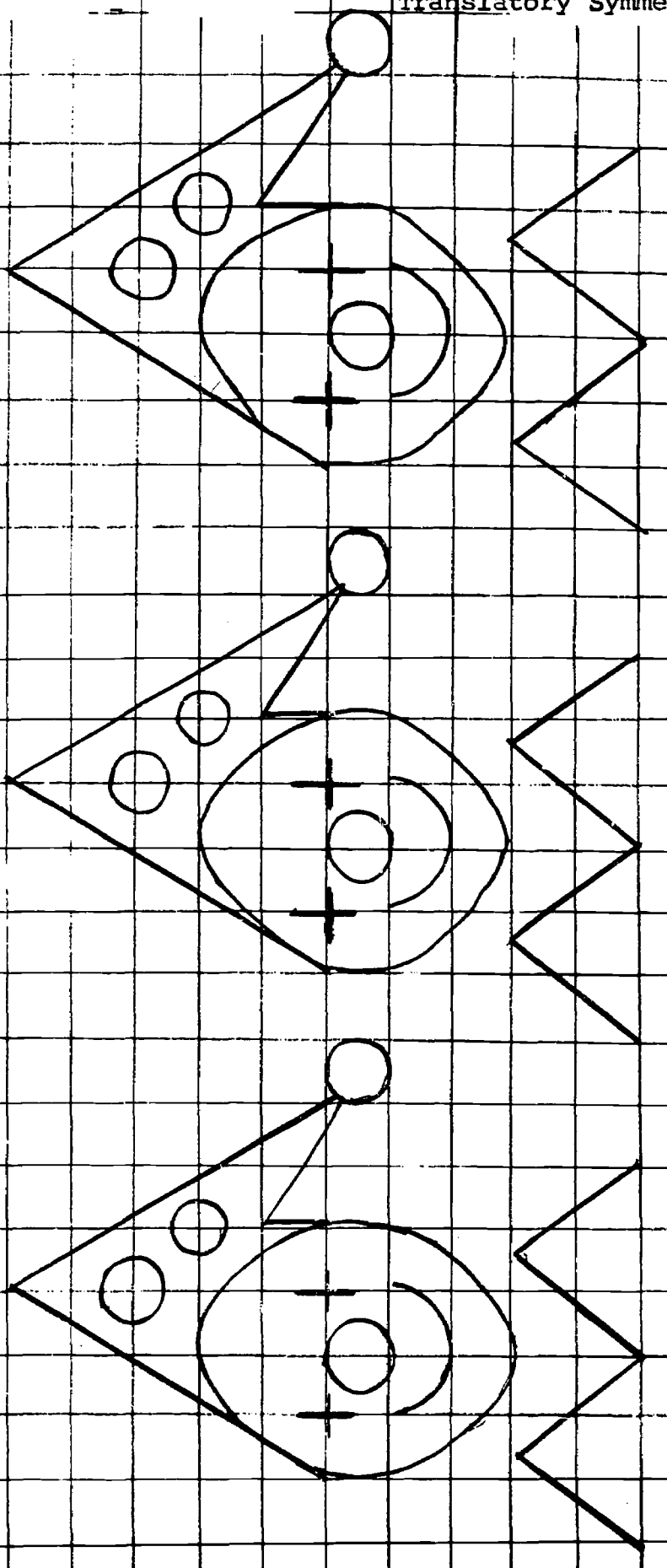


Translatory Symmetry

Design B



Design C



Commentary on Worksheet 4 - Design D - Repeating Patterns

We see one pattern on the Design D paper.

Trace this pattern on your tracing paper.

Now move the tracing paper to the left so that the open curved line and the circle on the tracing paper fit in the square to the left of the pattern.

Trace the pattern again.

Move your tracing paper until Design D is finished on your tracing paper.

Commentary on Worksheet 5 - Design E - Repeating Patterns

Look, boys and girls!

Where is Design E?

No, the artist didn't forget to put it in.

He said that it's your turn to make a design of your very own.

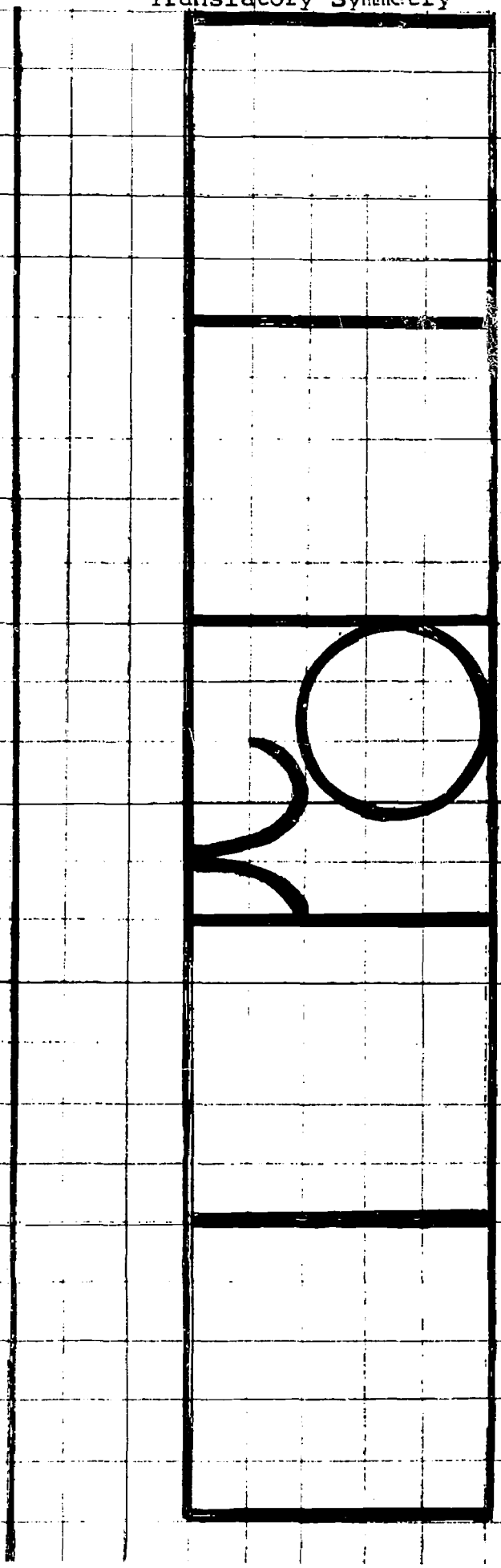
First of all, let's trace the five squares on the design paper.

How would you begin to make your design? (Ans.: Make a pattern in one box on the design paper, trace it, and repeat it in both directions.)

Fill your first box with any line pattern you think will make a pretty design.

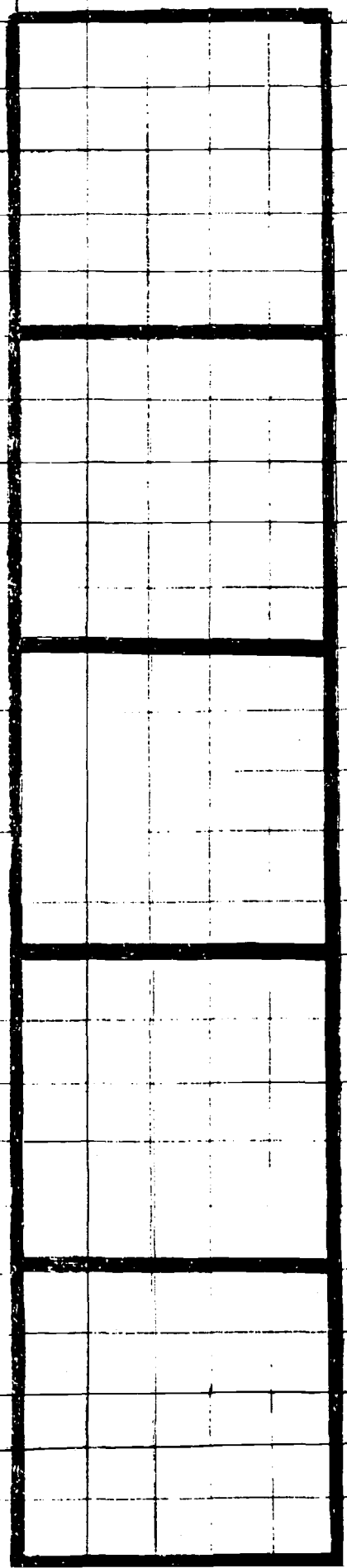
Translatory Symmetry

Design D



Translatory Symmetry

DESIGN E



Additional Activity

The function of art as an educational experience is vital as an integral part of the education process. Art intensifies the learning situation in that it brings the "thought" back to its place of origin - our senses. The children may be able to hold their "simple translation thoughts" longer, imagine their relation to other things, and thus their true significance through an art experience at this time.

Even though choices and decisions may be made quickly, much of the following art activity is independently predetermined and contemplative, thus providing for some self-expression. With the translation goals in mind, however, the optimum in self-expression is necessarily prevented.

MATERIALS NEEDED:

12" by 18" manila, white, or pastel colored construction paper.
Tempera paint- two "in season" colors (thick consistency)
Paint pans
Newspaper or newsprint for work area protection

Two or three items for each child with edges or textures suitable for making prints. Start a collection several days in advance to be assured of having a sufficient number of objects. Possible items to be used: spools, jar covers, old lipstick cases, buttons may be used with paper clip handles, sponge pieces, old pencils, etc.

PROCEDURE:

1. The paper is to be folded to make eight boxes on each side. (Teacher demonstration and explanation of halves desirable.)
   a. Folded once in half the long way.
   b. Folded twice in half the short way.

2. Teacher demonstration of art project...
   a. Complete one design row using two or three items (i.e., spools, sponges, etc.)
   b. Show variations with color and objects.
   c. Verbally reinforce idea that the pattern is repeated in color as well as in order.
   d. Both rows may have the same pattern or the second row may be another pattern.

3. The children choose two or three objects to make the first pattern in a box.

The products of this lesson make an attractive bulletin board alone or combined with pictures and items illustrating translatory symmetry.

There are innumerable variations of this type of art activity which might be more applicable to your classroom: e.g., potato prints, linoleum prints, color crayon prints, textile prints.

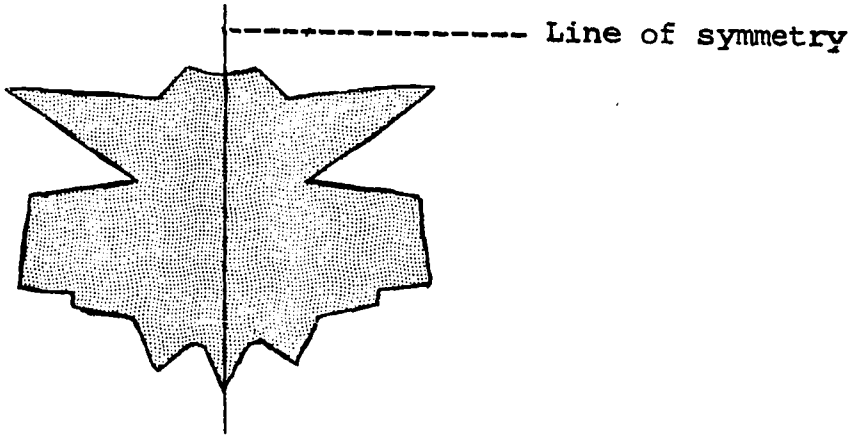
UNIT VI

BILATERAL SYMMETRY

Teacher Background

Operational Definitions (to be used in classroom discussions):

Line of symmetry: The line that divides a design into two parts which are identical in size and shape, one part being the mirror reflection of the other.



Symmetrical design or pattern: A design or pattern which has at least one line of symmetry.

Suggested Activities

Song: "Looby Loo", Music for Young Americans, A.B.C. Music Series, Kdgn., American Book Company, New York: 1959, p.83. (The class sings through "Looby Loo" with the corresponding motions as a review of left and right conception.)

TEACHER: There are several lefts and rights in our song. Do we have any more lefts and rights that we could add to our song? Should we sing, "I put my right head in"? Why not? Let's see how many rights and lefts we could add to the song. (Instruct the children to touch the proper parts as they are listed. Possible responses: eyebrows, eyes, ears, nostrils, shoulders, elbows, wrists, hands, hips, knees, ankles, feet.)

Experiments in Bilateral Symmetry:

TEACHER: Put both your hands, thumbs side by side, flat on your desk. Look at your left hand. What comes first going from left to right? (Little finger.)

Look at your right hand. What comes first going from left to right? (Thumb.)

Are your thumbs and fingers in the same order from left to right in both hands? (No, they are in opposite order.)

That's correct - just as day is the opposite of night, left is the opposite of right.

What do you see when you look in a mirror? (A mirror reflection.)

Commentary on Worksheet 6 - Design 1 - Mirror Reflection

Draw Design 1 on the chalkboard using a two inch to one square ratio.

Let's trace Design 1 with our pointing finger.

Find the line in the design that you think divides the design in half. Put your pointing finger on the line. (Check each child... Instruct one of the children who has the correct line to go to the board and show it on the big design on the board.)

Let's see if the two halves of the design are the same size. Count all the squares inside the left half of Design 1. How many? (18) How many squares are in the right half? (18) Are the two halves the same size? (Yes.)

Now let's see if the half of the design to the left of the dividing line is the same shape as the right half, but in opposite order just as our fingers and thumbs were when we used the mirror.

> Note: To prove the hypothesis that both halves are identical have the children do the following:
> 1. Trace Design 1 on your tracing paper including the dividing line.
> 2. Fold the tracing paper along the dividing line.
> 3. Holding the folded tracing paper tightly, turn it toward the light. As the light shines through the tracing paper, the design can be seen.

Are the two halves of the design the same? (Yes)

How can you tell? (They fit exactly on top of one another.)

DESIGN 1 (Use of Mirror)

Take your mirror and hold it on the dividing line on Design 1 so that you can see only the left half of the design.

Hold the mirror straight up and down to the paper.

What do you see? (A mirror reflection of the left half of the design.) What do you notice about the reflection? (It's the same size and shape only in opposite order.)

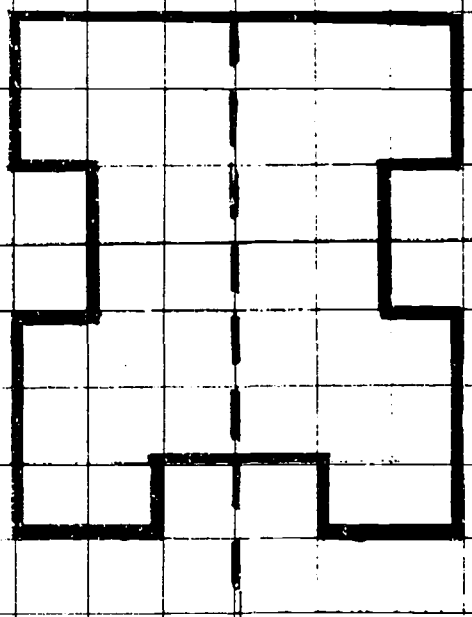
The line we placed our mirrors on is very important in many patterns and designs. This line is called the line of symmetry. It is the line that divides a design into two parts which are identical in size and shape with one part being the mirror reflection of the other part.

Put a pencil between your two thumbs, hands flat on your desk. What is the line of symmetry in the design made by your two hands and the pencil? (The pencil)

Worksheet 6

Bilateral Symmetry

DESIGN 1



Commentary on Worksheet 7 - Design 2 - Mirror Reflection

Does Design 2 have a line of symmetry? (Yes)

Lay your mirror flat on the right side of the line of symmetry. What do you think you'll see in the mirror when you hold it straight up and down on the line of symmetry? (The opposite of the left side, a mirror reflection.)

How many of you think Design 2 is a symmetrical design with both halves being identical in size and shape, one side being a mirror reflection of the other side?

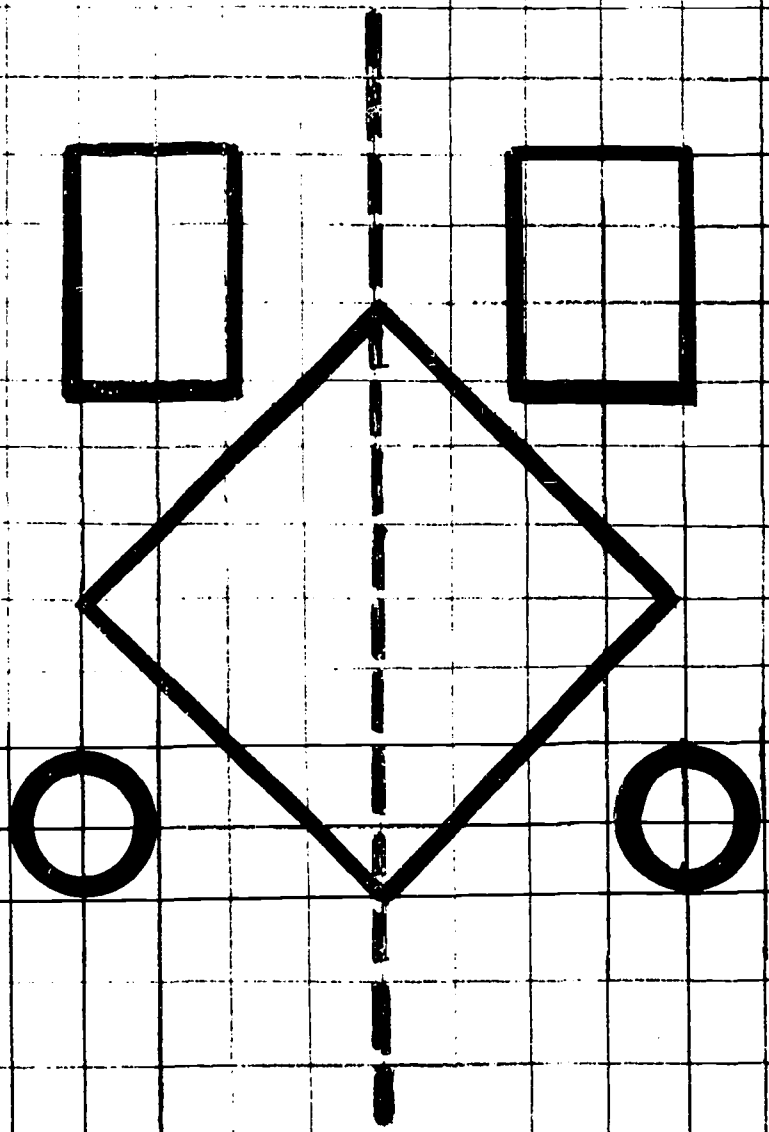
Let's take our tracing papers and see if we are correct.

Trace the design. (Be sure to include the line of symmetry.)

Fold the tracing paper design along the line of symmetry.

What did you find? (Both sides are identical in size and shape, with one side being the mirror reflection of the other.)

DESIGN 2



Commentary on Worksheet 8 - Design 3 - Mirror Reflection

Design 3 is just half finished.

Take your mirror and hold it on the dotted line. What will the pattern look like when the mirror reflection is made on the right?

Take your pencils and draw the mirror reflection on the right hand side.

1. How many squares long is the top line of the left side? (Five squares)
2. Start at the line of symmetry and on the same line go right five squares.
3. Now make a line on the right hand side that goes down as far as the line on the left. (Seven squares)
4. What line can we make easily now? (The bottom line)
5. You may put the door in now by going one square to the right from the line of symmetry opposite the line going to the left.
6. Finish the door reflection.
7. You may put the window in by yourself. Are there any questions? Raise your hand if you need help.

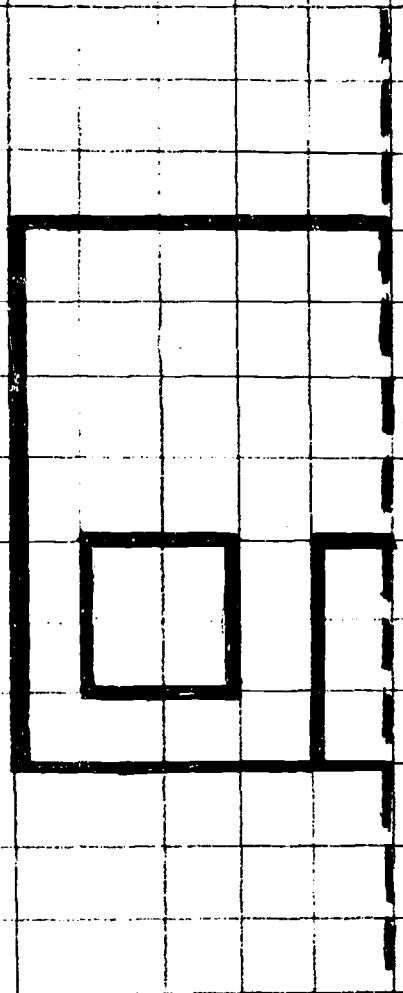
There is a way to check to see if you have made a perfect reflection. Take your tracing paper and trace Design 3, including the line of symmetry.

Fold your tracing paper along the line of symmetry.

Do both sides fit exactly together?

If they do, you made a perfect reflection.

DESIGN 3



Commentary on Worksheet 9 - Design 4 - Mirror Reflection

Design 4 consists of three exercises in mirror reflection. Use the same procedure as for Design 3. Remember to work from the line of symmetry in drawing the mirror reflection.

Commentary on Worksheet 10 - Design 5 - Mirror Reflection

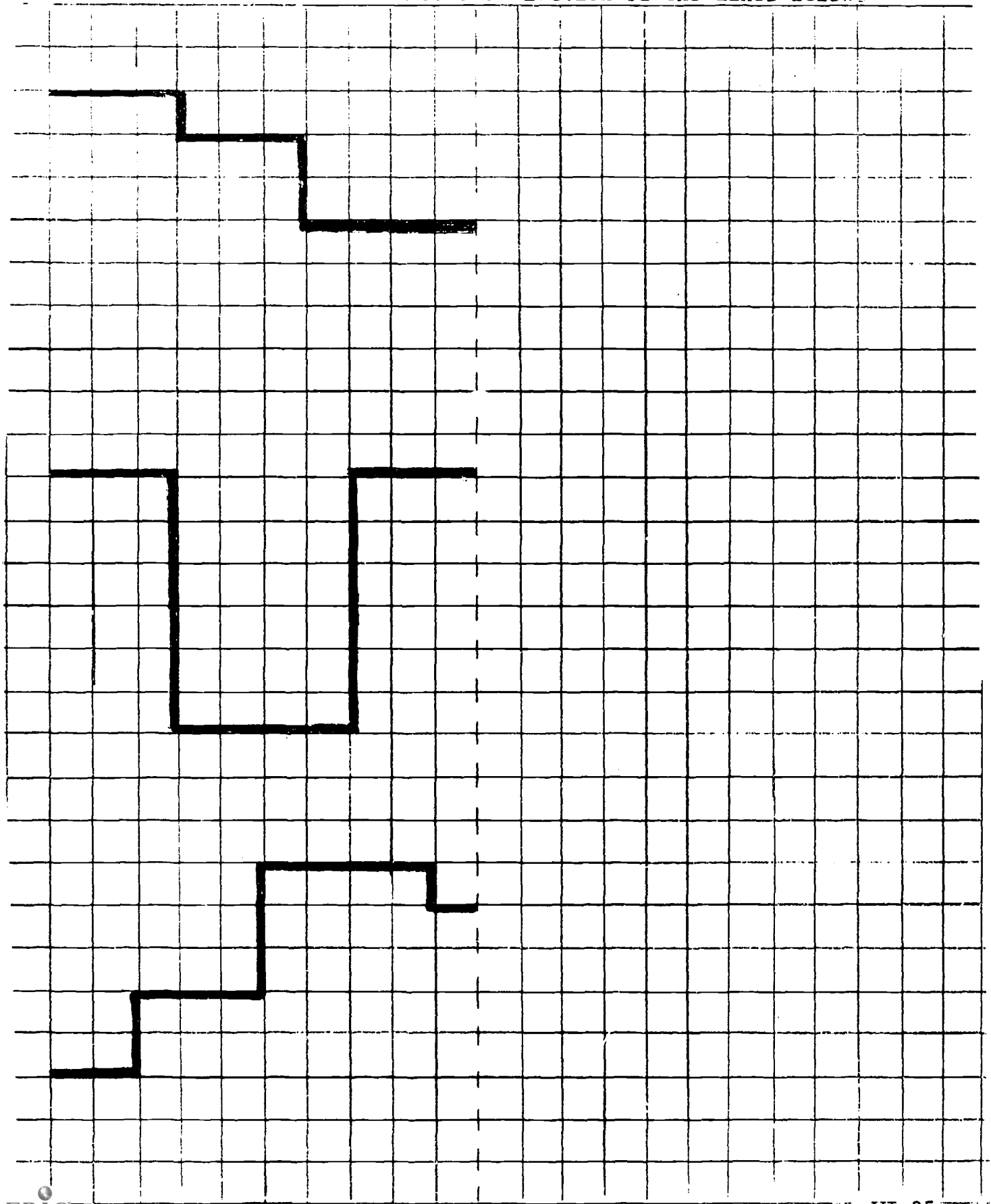
All that you see on Design 5 is the line of symmetry. You may make any design using squares. (Demonstrate on board.)

Use squares this time to be sure that the sizes and shapes will be the same on both sides of the line of symmetry. Remember that one side is in the opposite order of the other side. (Mirror reflection.)

Check to see if you made a perfect mirror reflection with your tracing paper.

DESIGN 4

Draw the mirror reflection of the lines below.

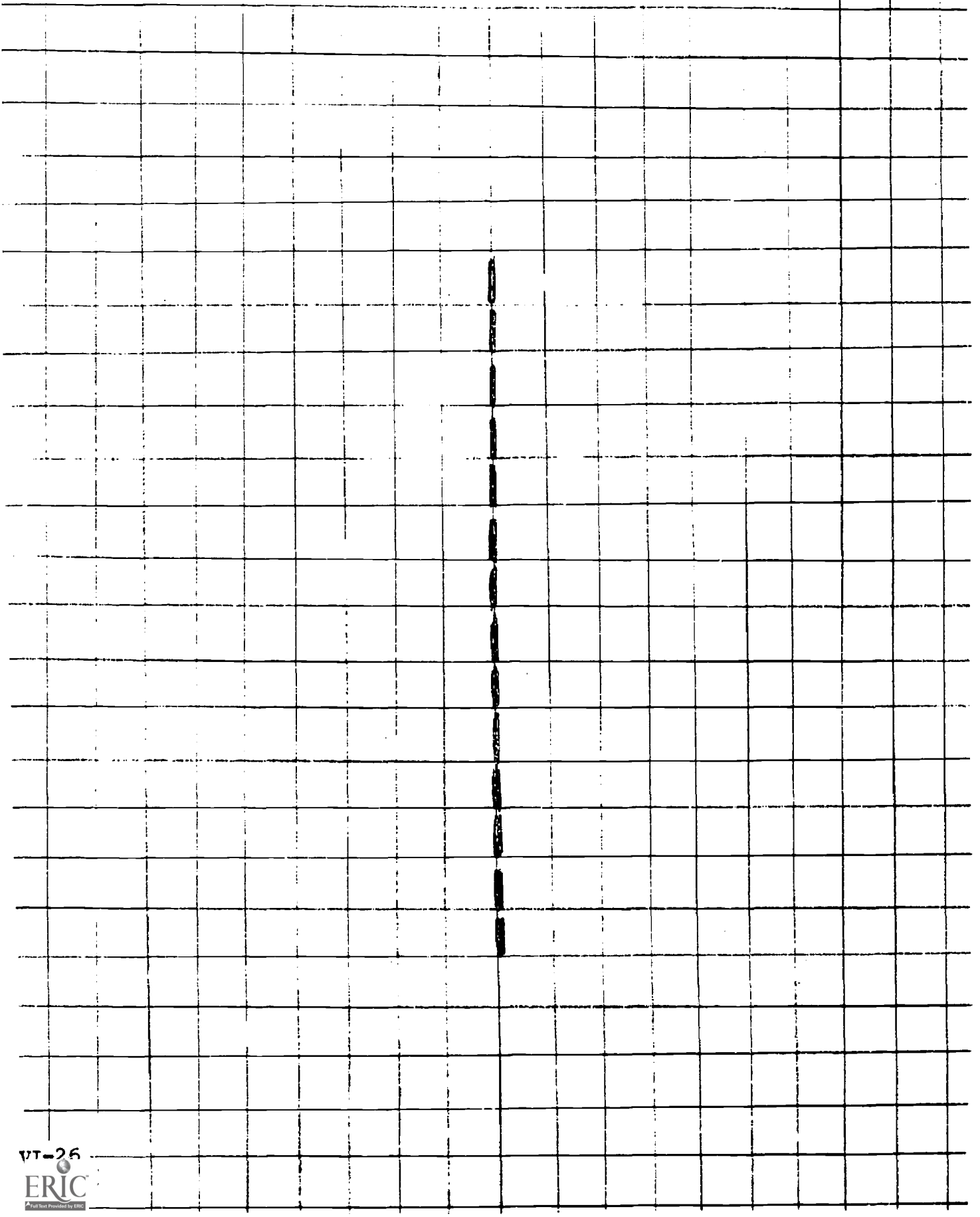


Worksheet 10

DESIGN 5

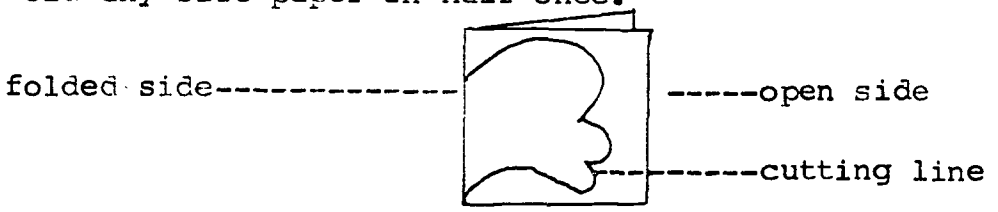
Bilateral Symmetry

Make a design of squares using mirror reflection.



Additional Activities on Bilateral Symmetry

1. Paper cutting activity:
   a. Fold any size paper in half once.



   b. Cut any shape as illustrated.
   c. Open; find line of symmetry. (Fold line)

2. Japanese String Painting:
   a. Fold construction paper in half once.
   b. Open construction paper.
   c. Drop string dipped in tempera paint on one half of the construction paper.
   d. Fold other half on top of string.
   e. Press down on the paper.
   f. Open the construction paper.
   g. Carefully lift the string.
   h. Find the line of symmetry. (Fold line)

3. Initiate a collection of pictures, objects, and names of things at school and at home which are examples of bilateral symmetry.

4. Flannelboard Game:
   a. The first child places one or more colored flannelboard shapes to the left of the line of symmetry made by a piece of string or yarn. Another child makes the mirror reflection to the right of the line of symmetry.

5. Floor Tiles:

It is possible that old sample tiles may be gotten without charge from local floor shops for symmetry experiments.

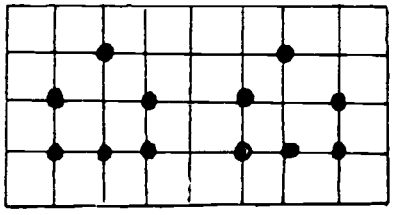
The most desirable tiles are those with a distinct design as compared with tiles which are randomly decorated.

It would be of use, if the tiles are not to be returned, to cut some in half diagonally and some in half horizontally. These pieces would lend variety to the types of patterns and designs which could be made.

6. Peg Board:

a. Symmetry may be illustrated on a peg board by using rubber bands to create line forms to be either repeated or reflected. Colored, heavy duty rubber bands are preferable . For example:

One child could place a set of pegs in the board. Another could repeat the pattern or reflect it. A third child could place the rubber bands around the two sets of pegs.



Peg Board

b. Colored yarn or string may be substituted for rubber bands in this activity.